

Lecture 30

Meyer's Theorem, Circuit Lower Bound

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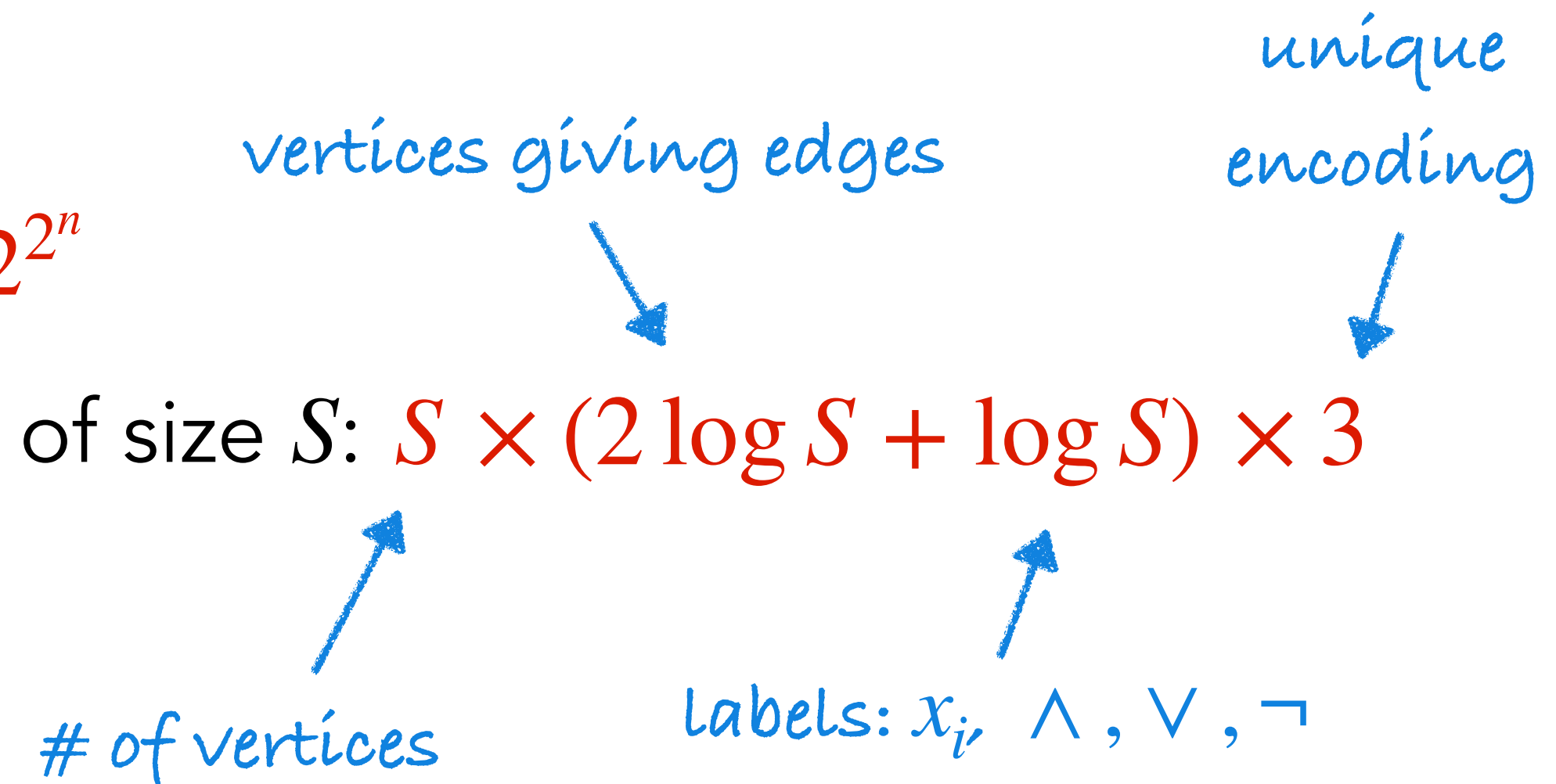
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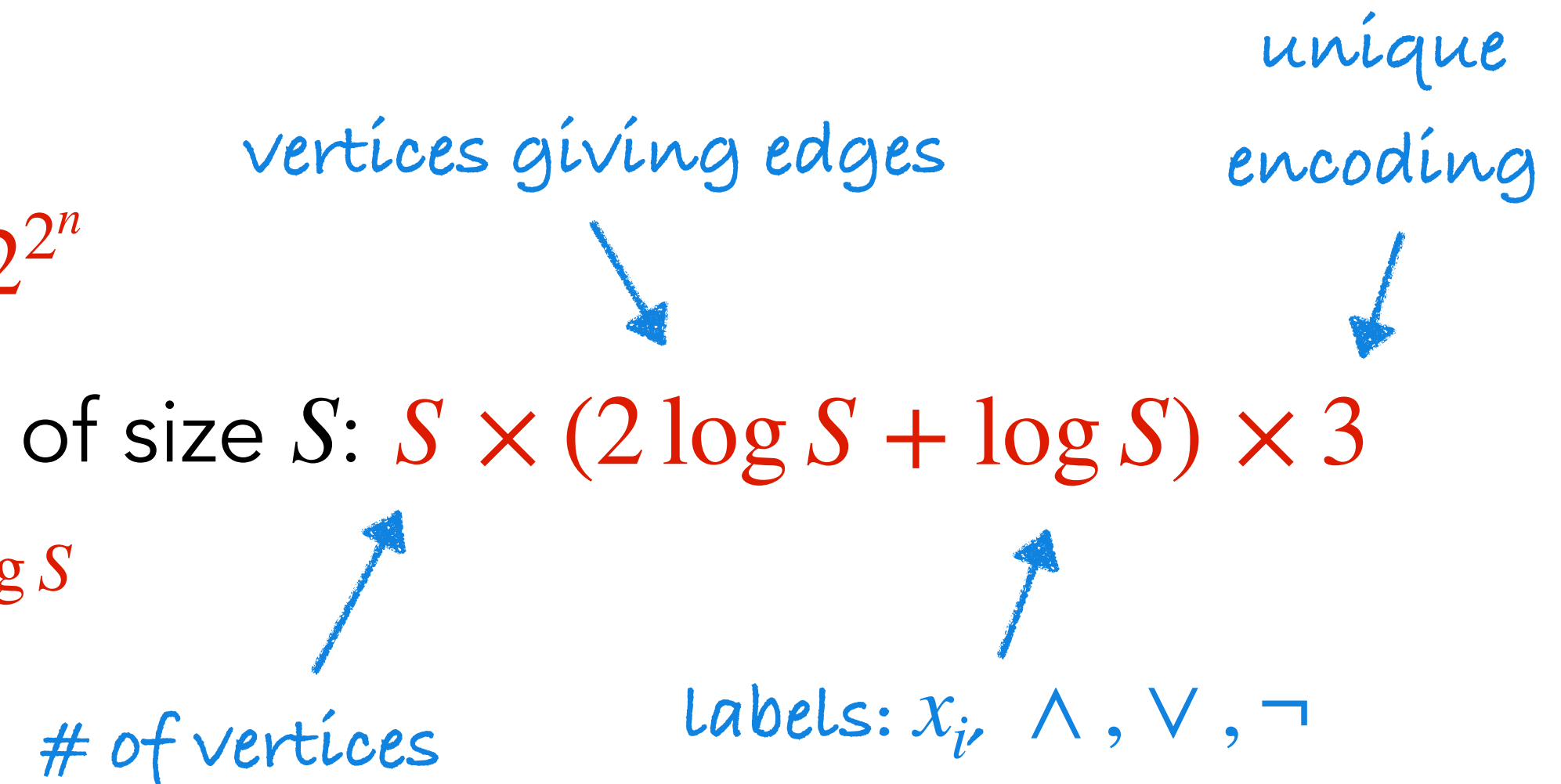


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$$2^{9S \log S} < 2^{9 \cdot 2^n / (10n) \cdot n}$$

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Labels: x_i, \wedge, \vee, \neg

Set $S = 2^n/(10n)$. Then, the number of circuits of size S is at most:

$$2^{9S \log S} < 2^{9 \cdot 2^n/(10n) \cdot n} < 2^{2^n}$$

Circuit Lower Bounds

Theorem: For every $n > 1$, there exists a function $f: \{0,1\}^n \rightarrow \{0,1\}$ that cannot be computed by a circuit C of size $2^n/(10n)$.

Proof: Counting argument:

- The number of functions from $\{0,1\}^n$ to $\{0,1\}$: 2^{2^n}
- The number of bits required to encode a circuit of size S : $S \times (2 \log S + \log S) \times 3$
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